

(DN) Draw isosceles triangle ABC with $AB = BC$ and isosceles triangle BCD with $BD = CD$ but neither BD nor CD is equal to AB . (Remember that each letter should only be used once in a diagram.)

Name _____ Per _____

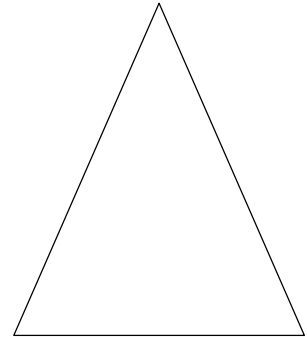
LO: I can use $SAS \cong$ to prove properties of isosceles triangles including the isosceles triangle theorem.

(1) **Congruence: Proving properties – base angles of an isosceles triangle**

transparencies, dry erase markers, eraser, compass, straightedge

(1) Prove: If a triangle is isosceles then the base angles are congruent.

(Add an auxiliary line that bisects the vertex angle.)



I know that . . .	because . . .

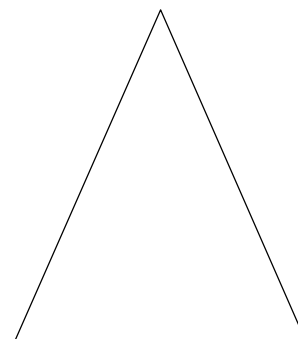
(1) **Congruence: Proving properties – Base angles of an isosceles triangle take 2**

cont.

Prove: If a triangle is isosceles then the base angles are congruent.

What does isosceles mean?

What are base angles?



① _____ because it is given.

(Label the triangle with letters and mark congruent parts based on definitions.)

② Side _____ \cong _____ because \triangle _____ is _____.

(What congruent parts did you mark?)

③ _____ bisects the vertex angle $\rightarrow \angle$ _____ because I can construct an auxiliary line segment.

(Where is the vertex angle? What does bisect mean? The segment needs a letter where it intersects the base of the triangle.)

④ \angle _____ \cong \angle _____ because segment _____ is the _____ of _____.

(Should you have congruent angles from step 3? Where? Why? Do you have to name them with 3 letters?

Mark this fact in the diagram.)

⑤ _____ \cong _____ because the segment is the same as itself (reflexive property)

(What segment is a side of both triangles? Mark this fact in the diagram.)

⑥ \triangle _____ \cong \triangle _____ because of _____ triangle congruence.

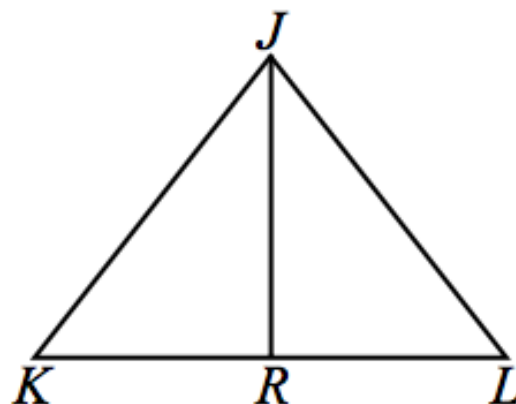
(Check your facts from steps 2, 4, and 5. Is that enough to prove the triangles are congruent? See lesson 5.1)

⑦ _____ \cong _____ because when triangles are congruent, all corresponding angle pairs and side pairs are _____. Since _____ and _____ are _____ and are _____ we can say that _____

(What were we trying to prove? What are the base angles?)

(2) **Congruence: Proving properties – Perpendicular**

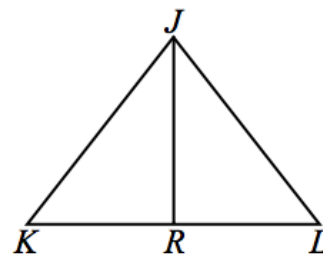
Given: $\overline{JK} \cong \overline{JL}$, \overline{JR} bisects $\angle KJL$ Prove: $\overline{JR} \perp \overline{KL}$



(2) Congruence: Proving properties – Perpendicular take 2

cont.

Given: $\overline{JK} \cong \overline{JL}$, \overline{JR} bisects $\angle KJL$ Prove: $\overline{JR} \perp \overline{KL}$



① I am given _____ AND _____.

(Mark congruent parts in the diagram.)

② \angle _____ \cong \angle _____ because _____ bisects _____.

(What was bisected? What does bisect mean? What congruent parts did you mark?)

③ _____ \cong _____ because the segment is the same as itself (reflexive property)

(What segment is a side of both small triangles? Mark this fact in the diagram.)

④ _____ \cong _____ because of _____ triangle congruence.

(Check your facts from steps 1, 2, and 3. Is that enough to prove the triangles are congruent? See lesson 5.1)

⑤ _____ \cong _____ because when triangles are congruent, all corresponding angle pairs and corresponding side pairs are _____.

(What were we trying to prove? What angles will help us do that?)

⑥ _____ + _____ = 180° because they are a linear pair.

(How can 180° help us get to 90° ?)

⑦ _____ + _____ = 180° by substitution

(How can we put the information from steps 5 and 6 together?)

⑧ $2(\text{_____}) = 180^\circ$ by combining like terms

(How can we use information from step 7 to help us get what we want for step 9?)

⑨ _____ = _____ $^\circ$ by _____

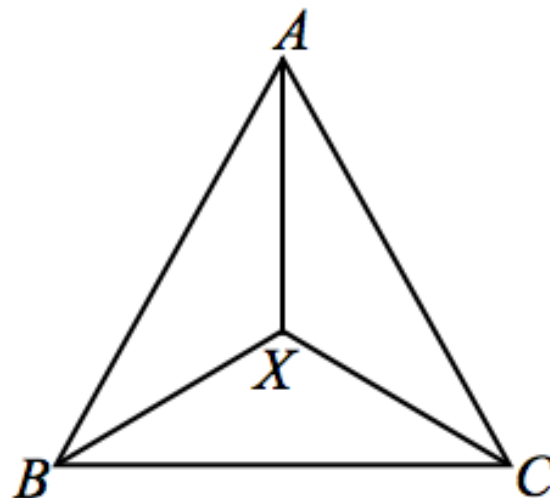
(What kind of angles will help us get what we want in step 10?)

⑩ _____ \perp _____ because lines are _____ when they intersect at _____ $^\circ$ angles.

(What were we trying to prove? What type of angles will help us do that?)

(3) Congruence: Proving properties – angle bisector

Given: $\overline{AB} \cong \overline{AC}$, $\overline{XB} \cong \overline{XC}$ Prove: \overline{AX} bisects $\angle BAC$

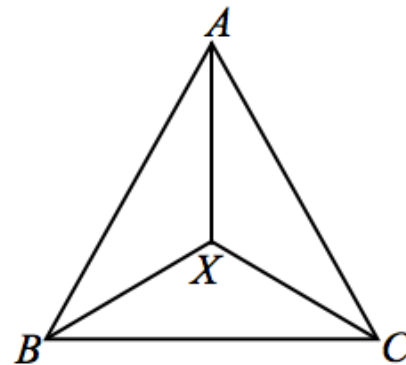


(3) Congruence: Proving properties – angle bisector take 2

cont.

Given: $\overline{AB} \cong \overline{AC}$, $\overline{XB} \cong \overline{XC}$

Prove: \overline{AX} bisects $\angle BAC$



① _____ AND _____ because it is given.

(Mark congruent parts.)

② $\angle ABC \cong \angle$ _____ and $\angle XBC \cong \angle$ _____ because base angles of _____.

(Refer to notes from lesson 4.5.)

③ $\angle ABX = \angle ABC - \angle$ _____ and $\angle ACX = \angle$ _____ - \angle _____
because _____

(What equations can we write with the angles we know?)

④ $\angle ABC - \angle$ _____ = \angle _____ - \angle _____

because _____

Therefore $\angle ABX = \angle ACX$

because _____

(What equations can we write with the angles we know?)

⑤ \triangle _____ $\cong \triangle$ _____ because _____.

(Which angles must be congruent to prove that?)

⑥ \angle _____ $\cong \angle$ _____ because when triangles are congruent, all corresponding angle pairs and corresponding side pairs are _____.

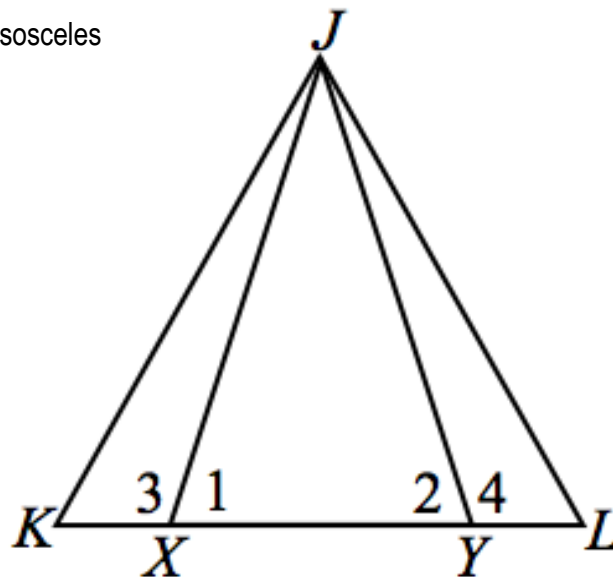
(What were we trying to prove? What angles will help us do that?)

⑦ \overline{AX} bisects \angle _____ because when 2 adjacent angles are congruent, the larger angle formed by the two angles must have been cut in half (bisected).

(What does bisect mean? What evidence will show us that an angle is bisected?)

(4) **Congruence: Proving properties – isosceles**

Given: $\overline{JX} \cong \overline{JY}$, $\overline{KX} \cong \overline{LY}$ Prove: $\triangle JKL$ is isosceles

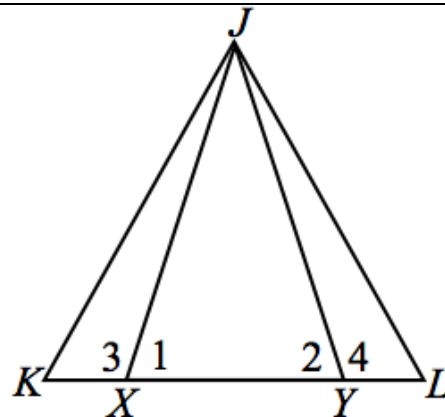


(4) **Congruence: Proving properties – isosceles take 2**

cont.

Given: $\overline{JX} \cong \overline{JY}$, $\overline{KX} \cong \overline{LY}$

Prove: $\triangle JKL$ is isosceles



① I am given _____ AND _____.

(Mark congruent parts.)

② \angle _____ \cong \angle _____ because base angles of an _____
are _____.

(Refer to notes from lesson 4.5.)

③ \angle _____ + \angle _____ = _____ and \angle _____ + \angle _____ = _____
because _____

(What angle relationships do you see with angles 1, 3, 2, and 4?)

④ \angle _____ + \angle _____ = \angle _____ + \angle _____ because _____

Therefore \angle _____ = \angle _____ because _____

(Since angles 1 and 2 are equal, what can we do to simplify the equation? Mark the diagram.)

⑤ \triangle _____ \cong \triangle _____ because _____.

(Which pair of triangles should we prove congruent to get the congruent segments we want?)

⑥ _____ \cong _____ because when triangles are congruent, all corresponding angle pairs and
corresponding side pairs are _____.

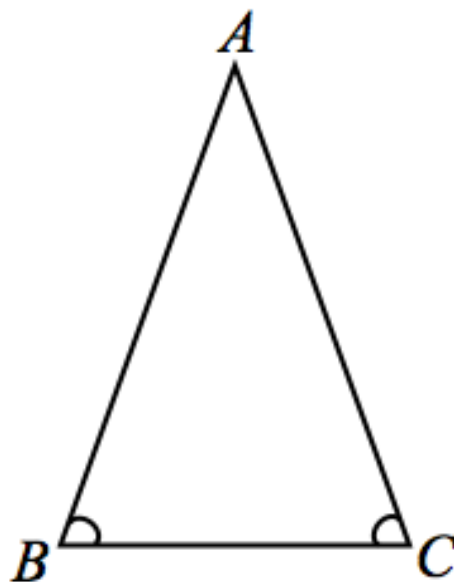
(What were we trying to prove? What segments will help us do that?)

⑦ $\triangle JKL$ is isosceles because it has _____ congruent sides, side _____ and side _____.

(What does isosceles mean? What evidence will show us that a triangle is isosceles?)

(5) **Congruence: Proving properties – equal sides**

Given: $\triangle ABC$ with $\angle CBA \cong \angle BCA$ Prove: $\overline{BA} \cong \overline{CA}$

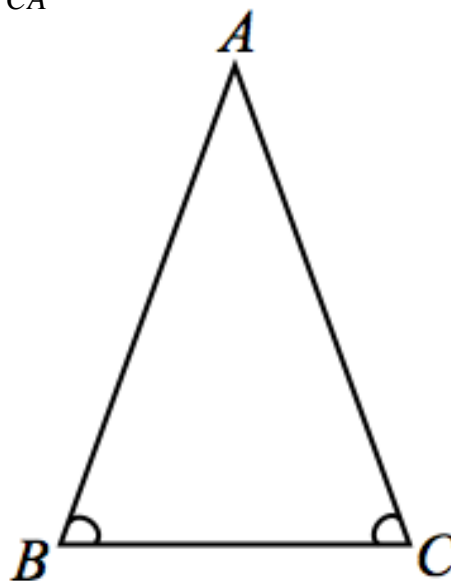


(5) Congruence: Proving properties – equal sides take 2

cont.

Given: $\triangle ABC$ with $\angle CBA \cong \angle BCA$

Prove: $\overline{BA} \cong \overline{CA}$



① I am given _____.

(Mark congruent parts.)

② Construct the perpendicular bisector of \overline{BC} and label the point of intersection D.

Now, \angle _____ = \angle _____ = _____° and

_____ \cong _____ because _____

(Refer to unit 2. What do we know when we have a perpendicular bisector? Mark this in your diagram)

③ When reflected across the perpendicular bisector, ray BA and ray _____ coincide because points B and _____ coincide and $\angle B$ and $\angle C$ are _____. Point A must be on the perpendicular bisector because the rays intersect at point _____ which means _____ must coincide with itself when the rays coincide. The only way for this to occur is if point A is on the _____.

(Does A have to be on the perpendicular bisector?)

④ _____ \cong _____ because the segment is the same as itself (reflexive property)

(What segment is a side of both triangles? Mark this fact in the diagram.)

⑤ \triangle _____ \cong \triangle _____ because _____.

(Which pair of triangles should we prove congruent to get the congruent segments we want?)

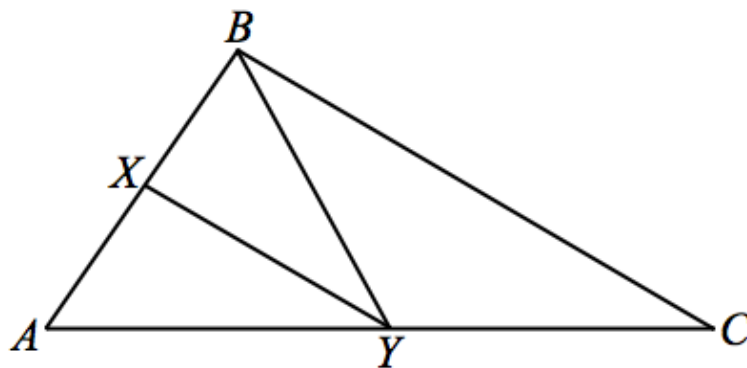
⑥ _____ \cong _____ because when triangles are congruent, all corresponding angle pairs and corresponding side pairs are _____.

(What were we trying to prove? Could we prove it if we had congruent triangles?)

□ (6) **Congruence: Proving properties – Base angles of an isosceles triangle take 2**

□ Given: $\triangle ABC$, \overline{XY} bisects $\angle BYA$ and $\overline{BC} \parallel \overline{XY}$,

Prove: $\overline{YB} \cong \overline{YC}$

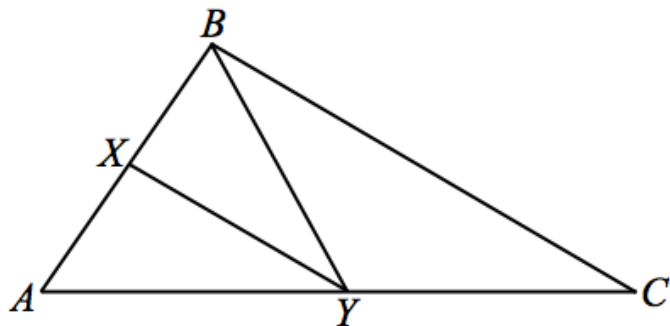


(6) **Congruence: Proving properties – Base angles of an isosceles triangle take 2**

cont.

Given: $\triangle ABC$, \overline{XY} bisects $\angle BYA$ and $\overline{BC} \parallel \overline{XY}$,

Prove: $\overline{YB} \cong \overline{YC}$



① I am given _____.

② \angle _____ \cong \angle _____ because _____.

(What does bisect mean? Mark the diagram.)

③ \angle _____ \cong \angle _____ because _____.

(What do we get out of parallel lines? Refer to notes from lesson 4.5. Mark the diagram.)

④ \angle _____ \cong \angle _____ because _____.

(What do we get out of parallel lines? Refer to notes from lesson 4.5. Mark the diagram.)

⑤ _____ \cong _____ because _____.

(What congruent angles will help us prove that the segments are congruent?)

⑥ _____ \cong _____ because _____.

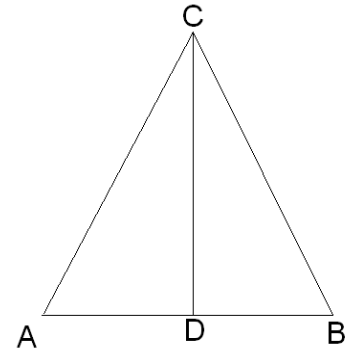
(Can we prove that segments are congruent? Can we show that base angles of a triangle are congruent so we can get congruent sides?)

□ (7) **Exit Ticket**

□ Given: \overline{CD} bisects $\angle BCA$

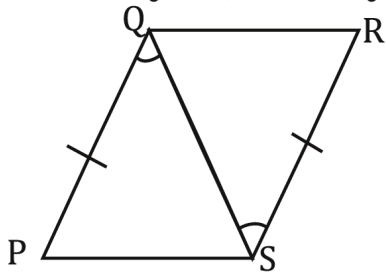
$$\overline{CA} \cong \overline{CB}$$

Prove: $\overline{AD} \cong \overline{BD}$



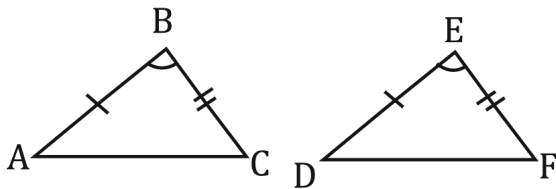
□ (8) **Homework**

(1) Given: $\overline{PQ} \cong \overline{RS}$, and $\angle PQS \cong \angle RSQ$



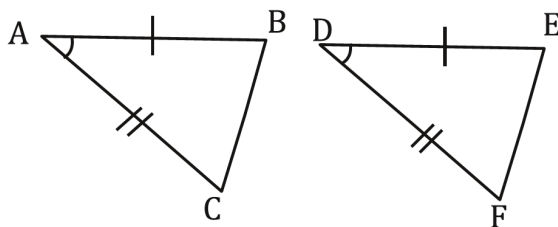
Prove: $\triangle ABC \cong \triangle DBC$

(2) Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle B \cong \angle E$



Prove: $\triangle ABC \cong \triangle DEF$

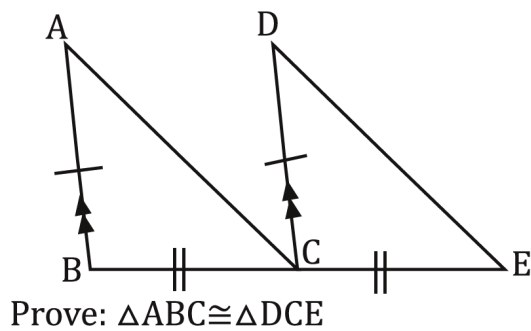
(3) Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$



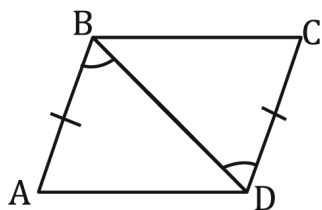
Prove: $\triangle ABC \cong \triangle DEF$

□ (8) Homework

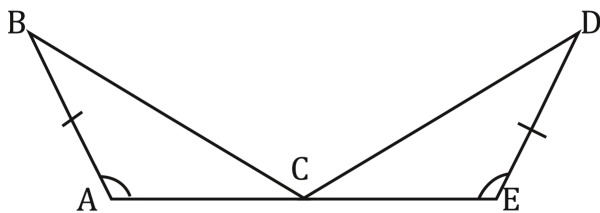
- (4) Given:
- $\overline{AB} \cong \overline{DC}$
- ,
- $\overline{AB} \parallel \overline{DC}$
- , and
- $\overline{BC} \cong \overline{CE}$



- (5) Given:
- $\overline{AB} \cong \overline{CD}$
- ,
- $\angle ABD \cong \angle CDB$

Prove: $\triangle ABD \cong \triangle CDB$

- (6) Given: C is the midpoint of
- \overline{AE}
- ,
- $\overline{BA} \cong \overline{DE}$
- , and
- $\angle A \cong \angle E$

Prove: $\triangle ABC \cong \triangle EDC$